



PRIMJER 3. – *MATLAB*

filtdemo

Prijenosna funkcija (IIR)

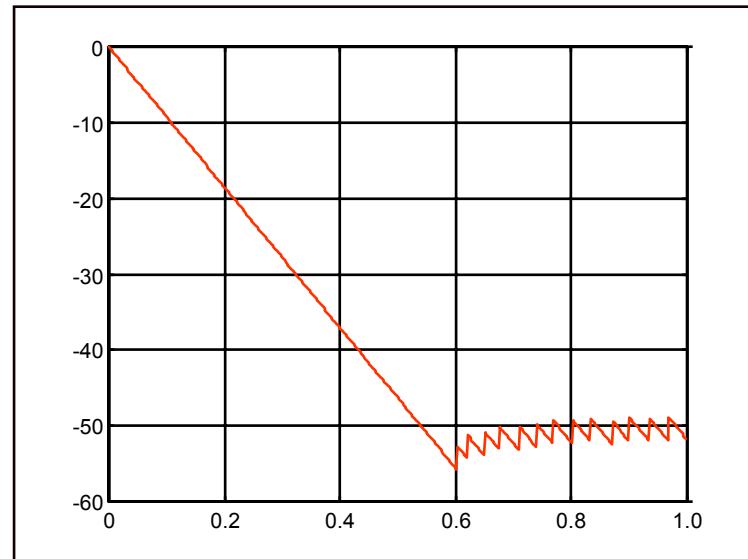
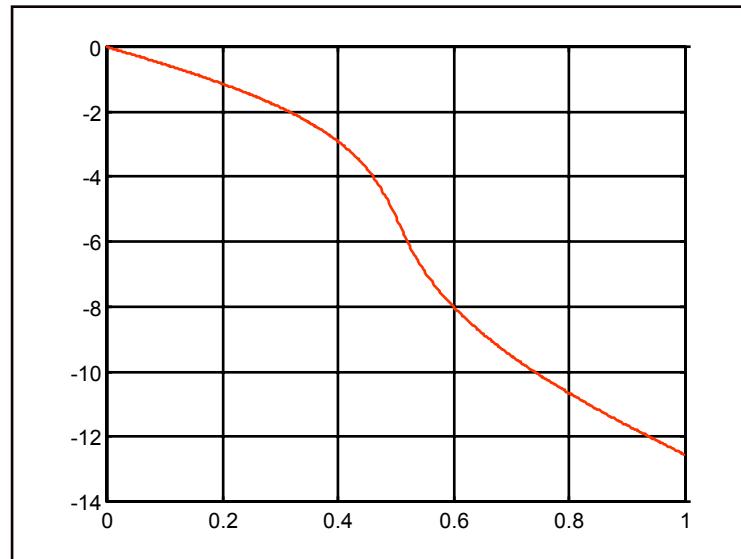
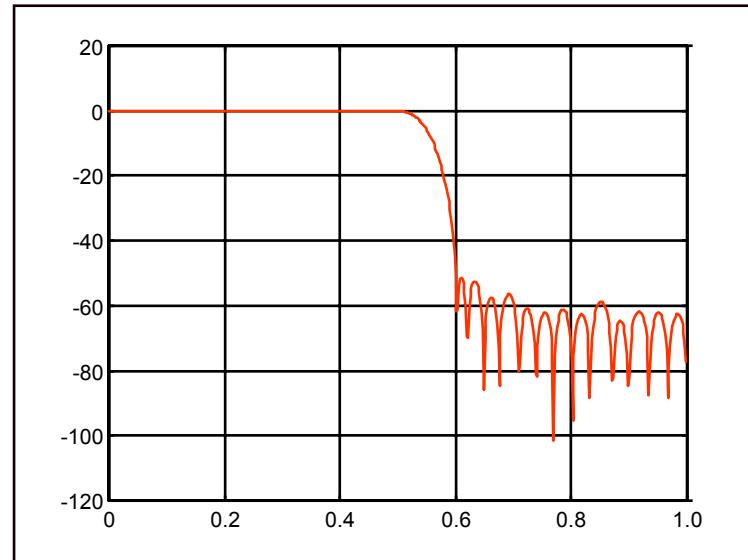
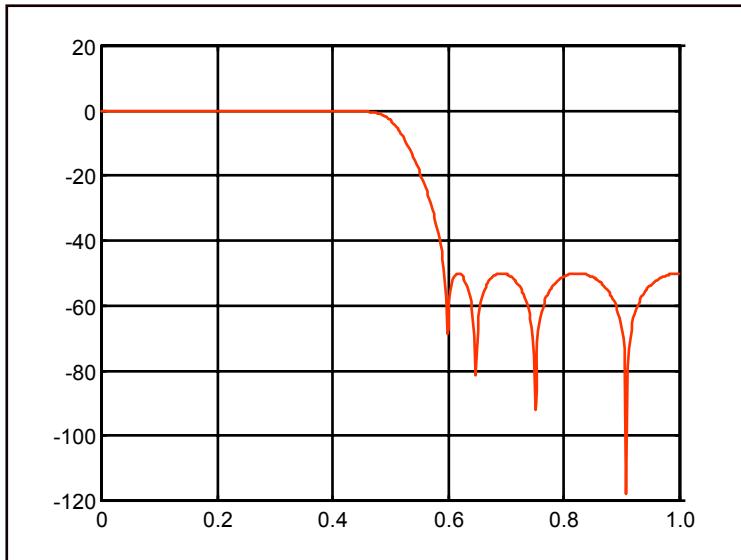
$$H(z) = \frac{0,06 + 0,30 \cdot z^{-1} + 0,78 \cdot z^{-2} + 1,30 \cdot z^{-3} + 1,53 \cdot z^{-4} + 1,30 \cdot z^{-5} + 0,78 \cdot z^{-6} + 0,30 \cdot z^{-7} + 0,06 \cdot z^{-8}}{1 - 0,95 \cdot z^{-1} + 1,74 \cdot z^{-2} + 1,20 \cdot z^{-3} + 0,92 \cdot z^{-4} + 0,41 \cdot z^{-5} + 0,15 \cdot z^{-6} + 0,03 \cdot z^{-7} + 0,004 \cdot z^{-8}}$$

Prijenosna funkcija (FIR)

$$H(z) = \sum_i b_i \cdot z^{-i}$$

1.	0.0004	21	-0.0176	41	-0.0152
2.	-0.0007	22	0.0270	42	0.0172
3.	-0.0004	23	0.0142	43	0.0067
4.	0.0015	24	-0.0432	44	-0.0148
5.	0.0002	25	-0.0042	45	-0.0010
6.	-0.0024	26	0.0673	46	0.0114
7.	0.0007	27	-0.0206	47	-0.0023
8.	0.0034	28	-0.1160	48	-0.0078
9.	-0.0023	29	0.1103	49	0.0038
10.	-0.0040	30	0.4839	50	0.0047
11.	0.0047	31	0.4839	51	-0.0040
12.	0.0038	32	0.1103	52	-0.0023
13.	-0.0078	33	-0.1160	53	0.0034
14.	-0.0023	34	-0.0206	54	0.0007
15.	0.0114	35	0.0673	55	-0.0024
16.	-0.0010	36	-0.0042	56	0.0002
17.	-0.0148	37	-0.0432	57	0.0015
18.	0.0067	38	0.0142	58	-0.0004
19.	0.0172	39	0.0270	59	-0.0007
20.	-0.0152	40	-0.0176	60	0.0004

Usporedba IIR / FIR

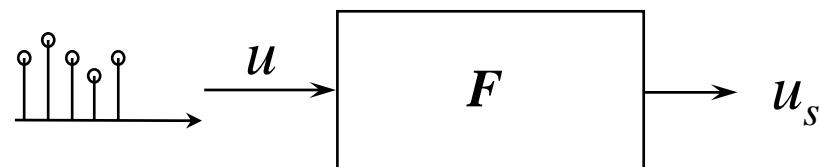




Nerekurzivni digitalni filtri

Primjer - aritmetička sredina

$$u_s = \frac{u_1 + u_2 + \dots + u_M}{M}$$



Interesantan je sustav koji služi za “glačanje” (usrednjavanje) slučajnih varijacija u signalu.

M-point moving average system

$$y[n] = \frac{u[n] + u[n-1] + u[n-2] + \dots + u[n-M+1]}{M} \quad \text{ili}$$

$$y[n] = \sum_{i=0}^{M-1} \frac{1}{M} \cdot u[n-i]$$

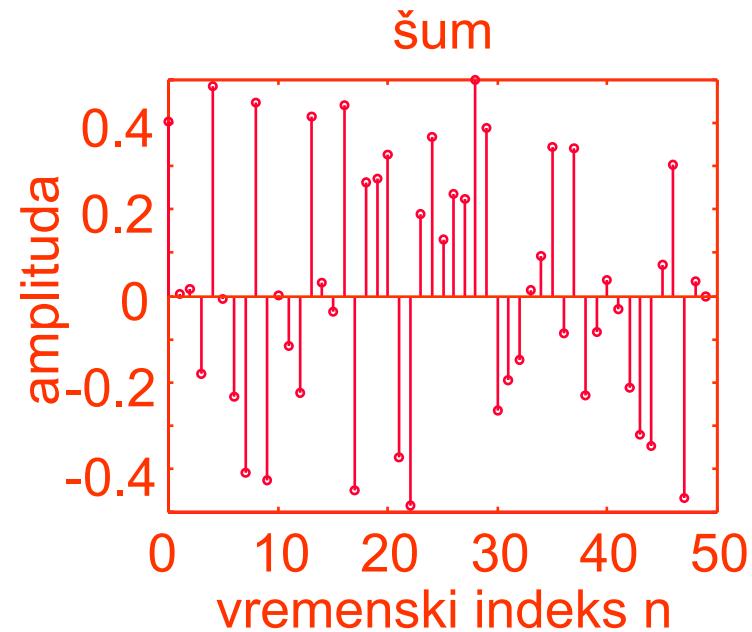
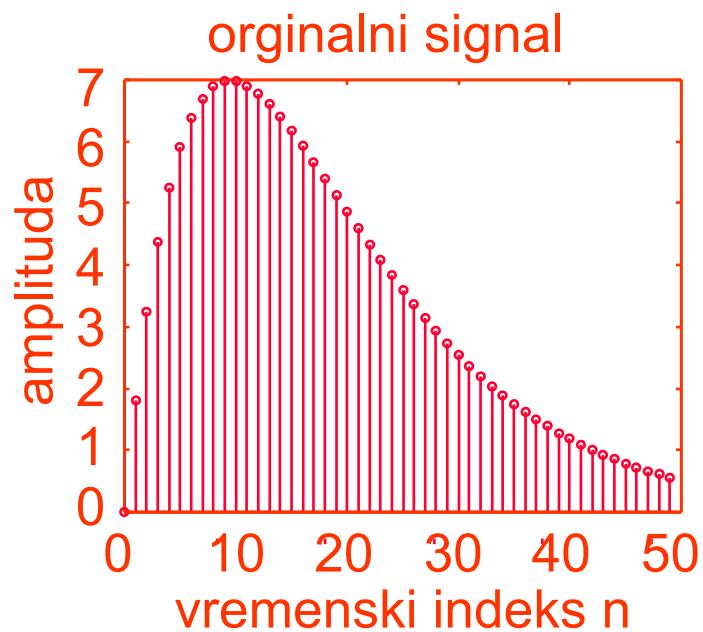
$$y[n] = \sum_{i=0}^{M-1} \frac{1}{M} u[n-i] = \sum_{i=0}^{M-1} h[i] \cdot u[n-i]$$

↔

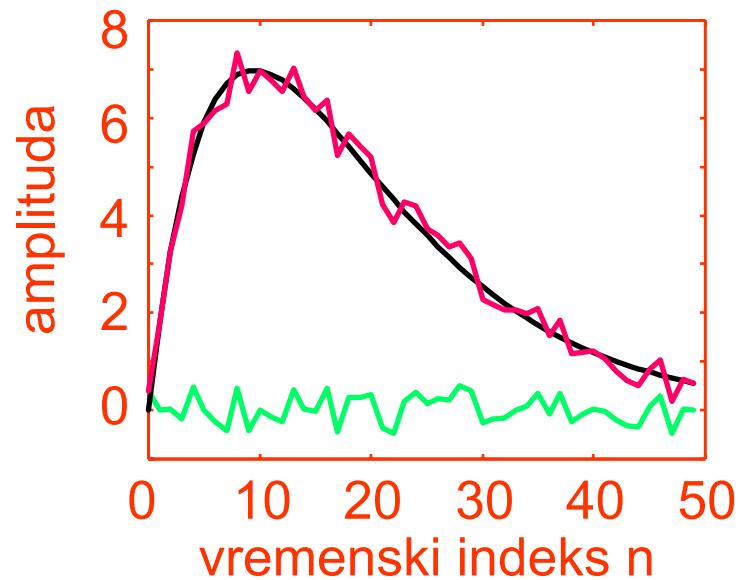
$$h[i]$$

Sustav ima konačan impulsni odziv - FIR sustav

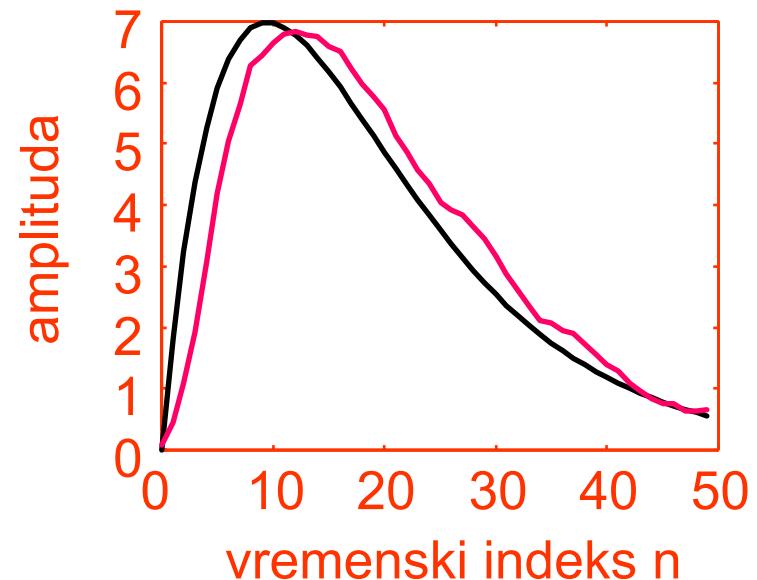
Primjer - *MATLAB*



Primjer - nastavak



- orginalni signal
- šum
- signal + šum



- orginalni signal
- $y[n]$ - izlaz iz *moving average* filtra

FIR sustav za $M=2$

Pogledajmo o kojem se sustavu radi:

Uzmimo $M=2$

$$y[n] = \sum_{i=0}^1 h[i] \cdot u[n-i] = \left| h[i] = \frac{1}{M} = \frac{1}{2} \right| =$$

$$= \sum_{i=0}^1 \frac{1}{2} \cdot u[n-i] = \frac{1}{2} \cdot \sum_{i=0}^1 u[n-i] =$$

FIR sustav za M=2 ...

$$y[n] = \frac{1}{2} \cdot \sum_{i=0}^1 u[n-i] = \begin{cases} u[n] = e^{j\omega n} \\ -\infty \leq n \leq \infty \end{cases}$$

$$= \frac{1}{2} \left(e^{j\omega n} + e^{j\omega(n-1)} \right) = \frac{1}{2} e^{j\omega n} \left(1 + e^{-j\omega} \right) =$$

$$= \frac{1}{2} \left(1 + e^{-j\omega} \right) \cdot e^{j\omega n}$$

$$H(e^{j\omega})$$

FIR sustav za M=2 ...

Isto pomoću Z-transformacije

$$Y(z) = \frac{1}{2} \left(1 + z^{-1} \right) \cdot U(z)$$


$$H(z) \rightarrow z = e^{j\omega} \rightarrow H(e^{j\omega})$$

FIR sustav za M=2 ...

$$H(e^{j\omega}) = \frac{1}{2} (1 + e^{-j\omega}) =$$

$$= \frac{1}{2} \left(e^{j\frac{\omega}{2}} \cdot e^{-j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}} \cdot e^{-j\frac{\omega}{2}} \right) =$$

$$= e^{-j\frac{\omega}{2}} \cdot \frac{1}{2} \left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}} \right) = \cos\left(\frac{\omega}{2}\right) \cdot e^{-j\frac{\omega}{2}}$$



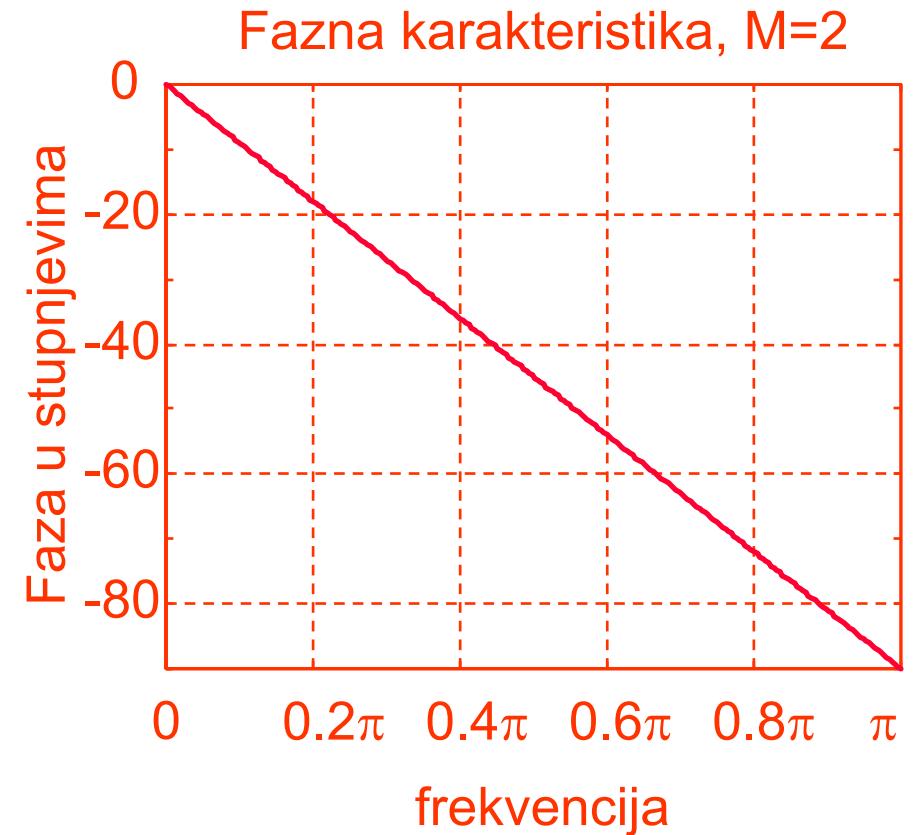
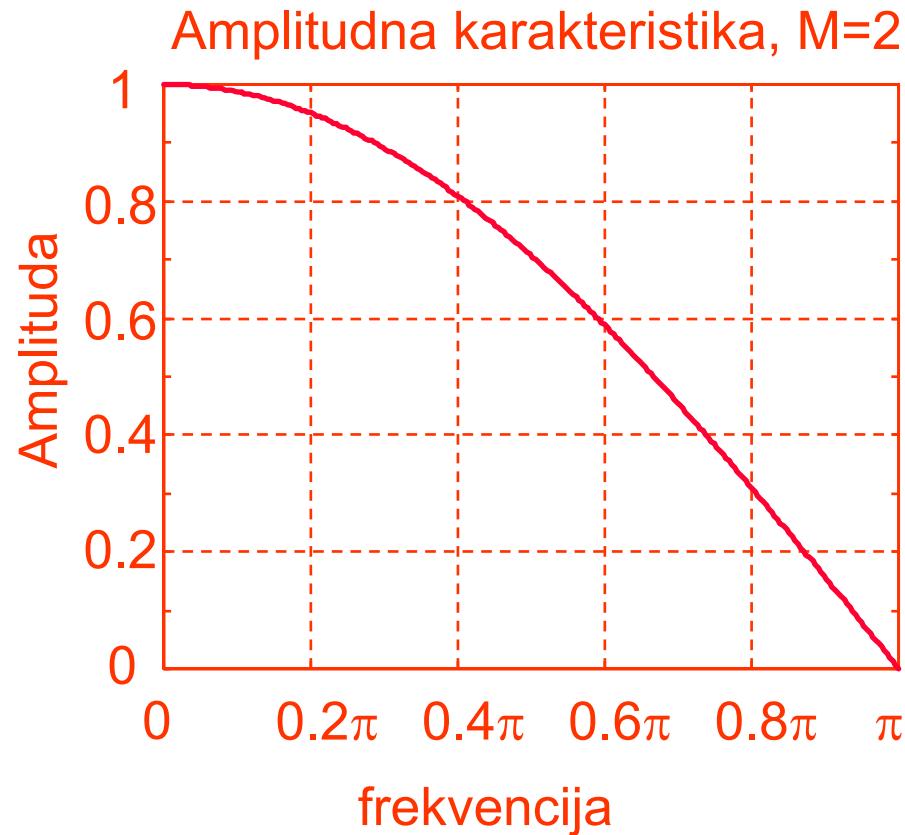
$$\cos\left(\frac{\omega}{2}\right)$$



Niskopropusni filter

FIR sustav za M=2 ...

$$H(e^{j\omega}) = \cos\left(\frac{\omega}{2}\right) \cdot e^{-j\frac{\omega}{2}}$$



FIR sustav za proizvoljni M

Za proizvoljni M vrijedi:

$$H(e^{j\omega}) = \sum_{n=0}^{\infty} h[n] \cdot e^{-j\omega n} = \left| h[n] = \begin{cases} \frac{1}{M}, & 0 \leq n \leq M-1 \\ 0, & \text{inače} \end{cases} \right| =$$

$$= \frac{1}{M} \sum_{n=0}^{M-1} e^{-j\omega n} = \dots$$

$$= \frac{1}{M} \cdot \frac{\sin \frac{M \cdot \omega}{2}}{\sin \frac{\omega}{2}} e^{-j(M-1)\frac{\omega}{2}}$$

FIR sustav za proizvoljni M ...

$$H(e^{j\omega}) = \frac{1}{M} \cdot \frac{\sin \frac{M \cdot \omega}{2}}{\sin \frac{\omega}{2}} e^{-j(M-1) \cdot \frac{\omega}{2}}$$

Amplitudno-fazna
karakteristika

Amplitudna
karakteristika

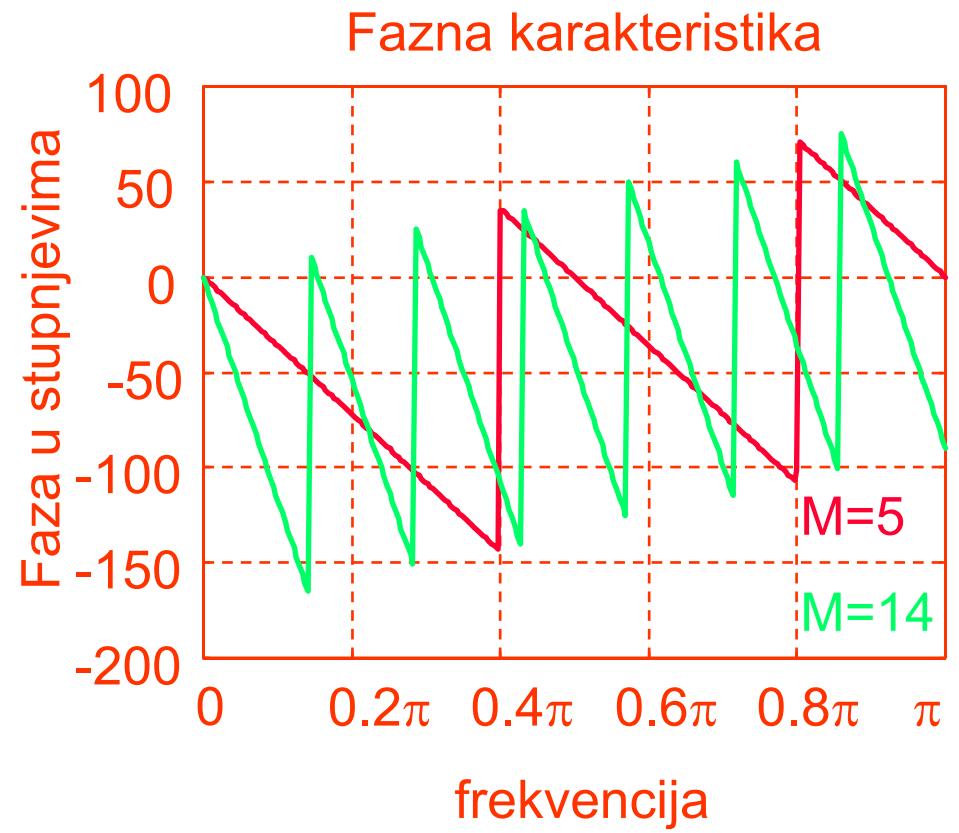
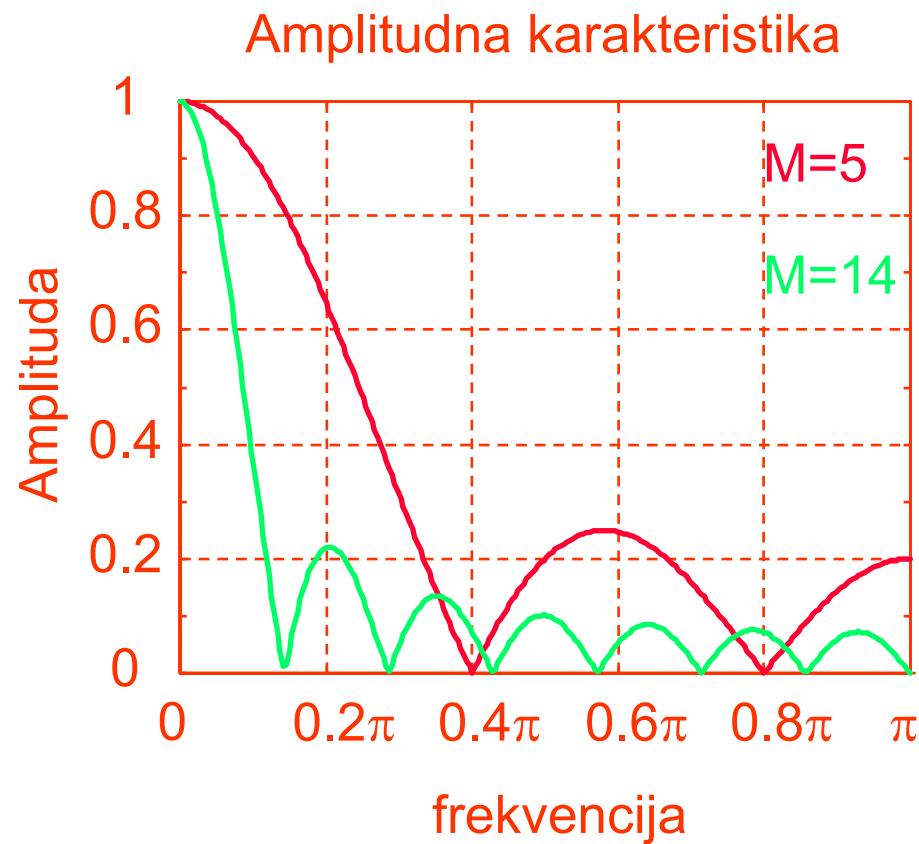
$$|H(e^{j\omega})| = \left| \frac{1}{M} \cdot \frac{\sin\left(\frac{M \cdot \omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)} \right|$$

Fazna
karakteristika

$$\theta(\omega) = -\frac{(M-1) \cdot \omega}{2} + \pi \cdot \sum_{i=0}^{\left\lfloor \frac{M}{2} \right\rfloor} \mu\left(\omega - \frac{2\pi i}{M}\right)$$

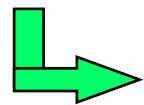
gdje je $\mu(\omega)$ step u ω

FIR sustav za proizvoljni M ...



Grupno kašnjenje

Daljnji parametar za karakterizaciju filtara je
grupno kašnjenje.



mjera linearnosti fazne funkcije

$$\tau(\omega) = -\frac{d\theta_c(\omega)}{d\omega}, \text{ gdje je } \theta_c(\omega) \text{ faza}$$

za prije navedeni primjer:

$$\tau(\omega) = \frac{M-1}{2}$$

Projektiranje FIR filtra

Prepostavimo signal koji je suma kosinusnih signala

$$u[n] = u_1[n] + u_2[n] = \{\cos(0,1 \cdot n) + \cos(0,4 \cdot n)\} \mu(n)$$

Prepostavimo da želimo sustav (filtrar) koji će:

- gušiti signal $u_1[n]$ (kosinus kutne frekv. 0,1 rad/sec)
- propuštati signal $u_2[n]$ (kosinus kutne frekv. 0,4 rad/sec)

Radi jednostavnosti uzmimo:

- red filtra $N=2$ (tri uzorka impulsnog odziva)
- impulsni odziv filtra

$$h[0] = h[2] = \alpha \quad \wedge \quad h[1] = \beta$$

Projektiranje FIR filtra ...

Jednadžba diferencija ovog sustava je:

$$\begin{aligned}y[n] &= h[0] \cdot u[n] + h[1] \cdot u[n-1] + h[2] \cdot u[n-2] = \\&= \alpha \cdot u[n] + \beta \cdot u[n-1] + \alpha \cdot u[n-2]\end{aligned}$$

a pripadna frekvencijska karakteristika

$$\begin{aligned}H(e^{j\omega}) &= h[0] + h[1] \cdot e^{-j\omega} + h[2] \cdot e^{-j2\omega} = \\&= \alpha \cdot (1 + e^{-j2\omega}) + \beta \cdot e^{-j\omega} = \\&= 2\alpha \cdot \left(\frac{e^{j\omega} + e^{-j\omega}}{2} \right) \cdot e^{-j\omega} + \beta \cdot e^{-j\omega} = \\&= (2\alpha \cdot \cos(\omega) + \beta) \cdot e^{-j\omega}\end{aligned}$$

Projektiranje FIR filtra ...

Amplitudna i fazna karakteristika filtra su

$$|H(e^{j\omega})| = 2\alpha \cdot \cos(\omega) + \beta$$

$$\theta(\omega) = -\omega$$

Iz zahtjeva na filter određujemo α i β

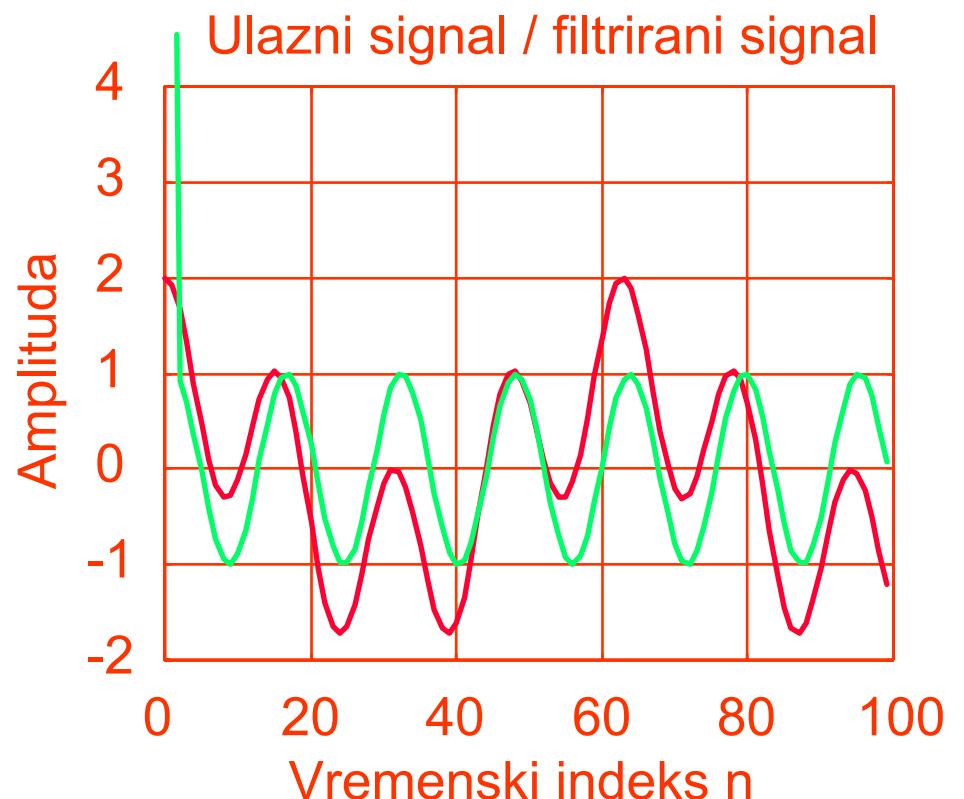
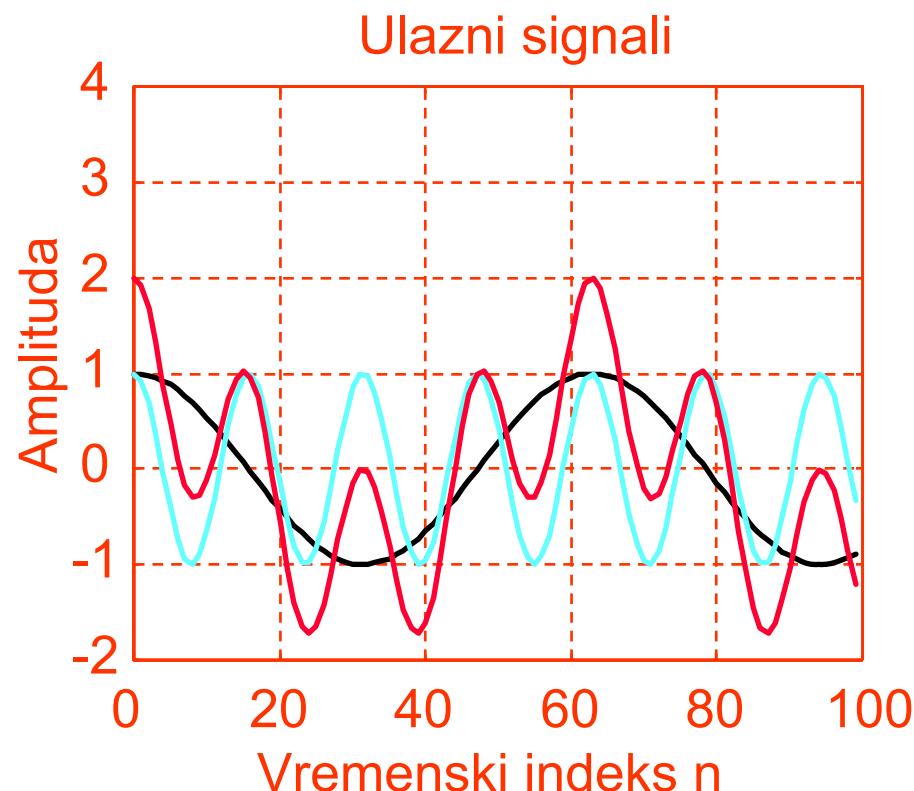
$$\left. \begin{array}{l} |H(e^{j \cdot 0,1})| = 2\alpha \cdot \cos(0,1) + \beta = 0 \\ |H(e^{j \cdot 0,4})| = 2\alpha \cdot \cos(0,4) + \beta = 1 \end{array} \right\} \begin{array}{l} \alpha = -6,76195 \\ \beta = 13,456335 \end{array}$$

što daje

$$y[n] = -6,76195 \cdot (u[n] + u[n-2]) + 13,456335 \cdot u[n-1]$$

Projektiranje FIR filtra ...

Uz pobudu: $u[n] = \{\cos(0,1 \cdot n) + \cos(0,4 \cdot n)\} \mu(n)$



FIR-filtri

- filtri s linearnom fazom
- simetrija impulsnog odziva je nužan preduvjet za linearu fazu
- ovisno o tipu simetrije impulsnog odziva definiramo četiri tipa FIR filtra sa realnim impulsnim odzivom duljine **N+1**

Tipovi FIR-filtara

- Tip 1 - simetričan impulsni odziv
 - neparan broj uzoraka impulsnog odziva
- Tip 2 - simetričan impulsni odziv
 - paran broj uzoraka impulsnog odziva
- Tip 3 - antisimetričan impulsni odziv
 - neparan broj uzoraka impulsnog odziva
- Tip 4 - antisimetričan impulsni odziv
 - paran broj uzoraka impulsnog odziva

Tip 1 FIR filtra

impulsni odziv zadovoljava slijedeći uvjet

$$h[n] = h[N - n], \quad 0 \leq n \leq N$$

za daljnje razmatranje pretpostavimo $N=8$. U tom slučaju prijenosna funkcija filtra je

$$\begin{aligned} H(z) = & h[0] + h[1] \cdot z^{-1} + h[2] \cdot z^{-2} + h[3] \cdot z^{-3} + h[4] \cdot z^{-4} + \\ & + h[5] \cdot z^{-5} + h[6] \cdot z^{-6} + h[7] \cdot z^{-7} + h[8] \cdot z^{-8} \end{aligned}$$

Tip 1 FIR filtra ...

Prema definiciji tipa 1 FIR filtra za $N=8$ vrijedi

$$h[0] = h[8], \quad h[1] = h[7], \quad h[2] = h[6], \quad h[3] = h[5],$$

te se prethodni izraz može pojednostaviti

$$\begin{aligned} H(z) &= h[0] \cdot (1 + z^{-8}) + h[1] \cdot (z^{-1} + z^{-7}) + \\ &\quad + h[2] \cdot (z^{-2} + z^{-6}) + h[3] \cdot (z^{-3} + z^{-5}) + h[4] \cdot z^{-4} = \\ &= h[0] \cdot z^{-4} \cdot (z^4 + z^{-4}) + h[1] \cdot z^{-4} \cdot (z^3 + z^{-3}) + \\ &\quad + h[2] \cdot z^{-4} \cdot (z^2 + z^{-2}) + h[3] \cdot z^{-4} \cdot (z + z^{-1}) + h[4] \cdot z^{-4} = \\ &= z^{-4} \cdot \{h[0] \cdot (z^4 + z^{-4}) + h[1] \cdot (z^3 + z^{-3}) + \\ &\quad + h[2] \cdot (z^2 + z^{-2}) + h[3] \cdot (z + z^{-1}) + h[4]\} \end{aligned}$$

Tip 1 FIR filtra ...

Pripadna frekvencijska karakteristika

$$H(e^{j\omega}) = e^{-j4\omega} \cdot \{2h[0] \cdot \cos(4\omega) + 2h[1] \cdot \cos(3\omega) + \\ + 2h[2] \cdot \cos(2\omega) + 2h[3] \cdot \cos(\omega) + h[4]\}$$

ili u općem slučaju

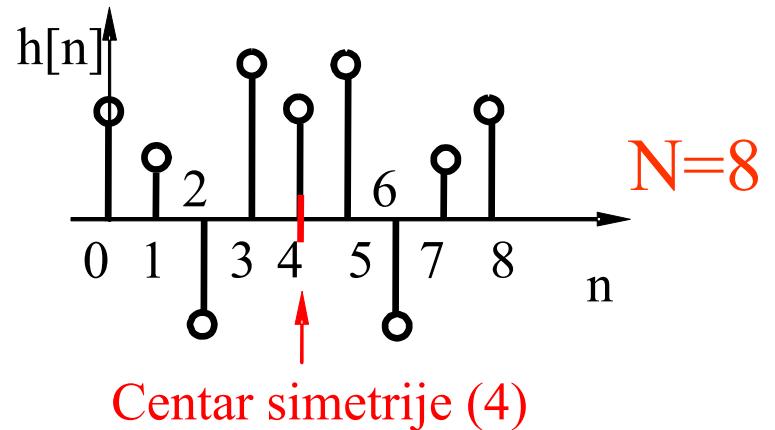
$$H(e^{j\omega}) = e^{-jN\omega/2} \cdot \left\{ \sum_{m=0}^{N/2} a[m] \cdot \cos(\omega \cdot m) \right\}$$

$$a[0] = h\left[\frac{N}{2}\right], \quad a[m] = 2h\left[\frac{N}{2} - m\right], \quad 1 \leq m \leq \frac{N}{2}$$

Tipovi FIR-filtara ...

Tip 1

- simetričan impulsni odziv
- neparan broj uzoraka



$$H(e^{j\omega}) = e^{-jN\omega/2} \cdot \left\{ \sum_{m=0}^{N/2} a[m] \cdot \cos(\omega \cdot m) \right\}$$

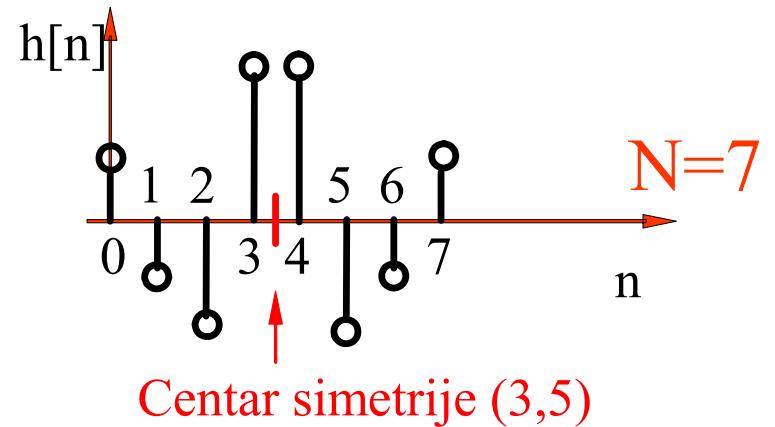
$$a[0] = h\left[\frac{N}{2}\right], \quad a[m] = 2 \cdot h\left[\frac{N}{2} - m\right], \quad 1 \leq m \leq \frac{N}{2}$$

$$\tau(\omega) = \frac{N}{2}$$

Tipovi FIR-filtara ...

Tip 2

- simetričan impulsni odziv
- paran broj uzoraka



$$H(e^{j\omega}) = e^{-jN\omega/2} \cdot \left\{ \sum_{m=1}^{(N+1)/2} b[m] \cdot \cos\left(\omega \cdot \left(m - \frac{1}{2}\right)\right) \right\}$$

$$b[m] = 2 \cdot h\left[\frac{N+1}{2} - m\right], \quad 1 \leq m \leq \frac{N+1}{2}$$

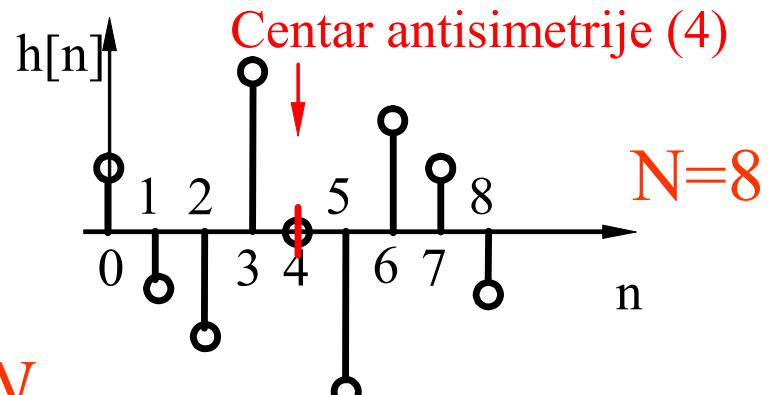
$$\tau(\omega) = \frac{N}{2}$$

Tipovi FIR-filtara ...

Tip 3

- antisimetričan impulsni odziv
- neparan broj uzoraka

$$h[n] = -h[N-n], \quad 0 \leq n \leq N$$



$$H(e^{j\omega}) = e^{-jN\omega/2} \cdot e^{j\pi/2} \cdot \left\{ \sum_{m=1}^{N/2} c[m] \cdot \sin(\omega \cdot m) \right\}$$

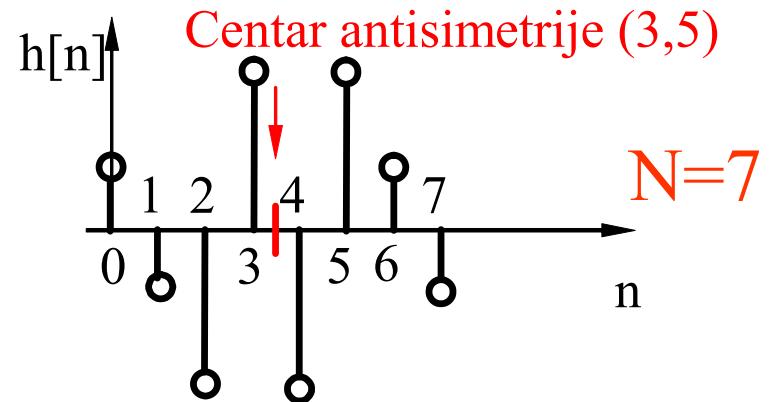
$$c[m] = 2 \cdot h\left[\frac{N}{2} - m\right], \quad 1 \leq m \leq \frac{N}{2}$$

$$\tau(\omega) = \frac{N}{2}$$

Tipovi FIR-filtara ...

Tip 4

- antisimetričan impulsni odziv
- paran broj uzoraka



$$H(e^{j\omega}) = e^{-jN\omega/2} \cdot e^{j\pi/2} \cdot \left\{ \sum_{m=1}^{(N+1)/2} d[m] \cdot \sin\left(\omega \cdot \left(m - \frac{1}{2}\right)\right) \right\}$$

$$d[m] = 2 \cdot h\left[\frac{N+1}{2} - m\right], \quad 1 \leq m \leq \frac{N+1}{2}$$

$$\tau(\omega) = \frac{N}{2}$$

Tipovi FIR-filtara ...

